22

A → purchase a cup of coffee

B → purchase a piece of cake

$P(A) = 0.7$

$P(B) = 0.4$

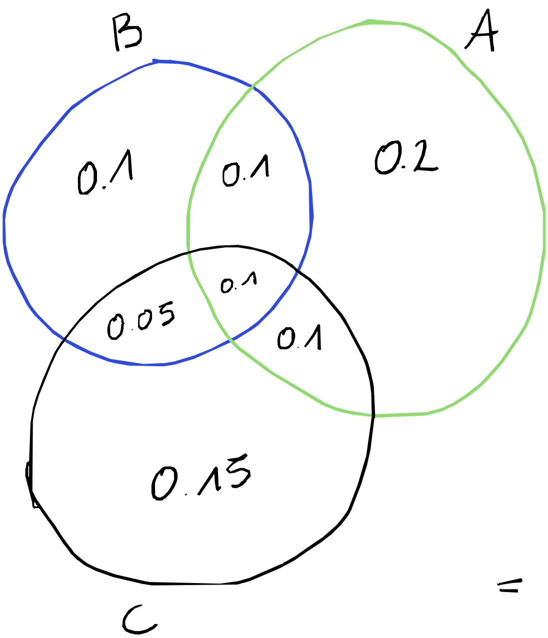
$P(A \cap B) = 0.2$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$$

(13)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} =$$

$$= \frac{0.2}{0.35} = \frac{2}{10} \cdot \frac{100}{35} = \frac{20}{35} = \frac{4}{7}$$



$$P(C|B) = \frac{P(B \cap C)}{P(B)} =$$

$$= \frac{0.15}{0.35} \quad \frac{15}{100} \cdot \frac{100}{35} = \frac{3}{7}$$

c) $$P(B|A \cup C) = \frac{P(B \cap (A \cup C))}{P(A \cup C)} = \frac{0.25}{0.7} = \frac{25}{100} \cdot \frac{100}{70}$$

$$= \frac{5}{14}$$

d) $$P(B|A,C) = \frac{P(B \cap A \cap C)}{P(A \cap C)} = \frac{0.1}{0.2} = \frac{1}{2}$$

(24)

(24)

a) $P(2 \leq X \leq 5) = \frac{5-2}{10} = \frac{3}{10}$

---

b) $P(X \leq 2 \mid X \leq 5) = \frac{P(X \leq 2 \cap X \leq 5)}{P(X \leq 5)} =$

$= \frac{P(X \leq 2)}{P(X \leq 5)} = \frac{0.2}{0.5} = \frac{2}{5}$

---

c) $P(3 \leq X \leq 8 \mid X \geq 4) =$

$= \frac{P(3 \leq X \leq 8 \cap X \geq 4)}{P(X \geq 4)} = \frac{P(4 \leq X \leq 8)}{P(X \geq 4)} =$

$= \frac{\frac{4}{10}}{\frac{6}{10}} = \frac{2}{3}$

(25)

$A \to$ "student got an A"

$B \to$ "student lived on campus"

$P(A) = \frac{120}{600} = \frac{2}{10} = \frac{1}{5}$ | $P(B) = \frac{200}{600} = \frac{1}{3}$ | $P(B^c) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(A \cap B^c) = \frac{80}{600} = \frac{2}{15}$

$P(A \cap B) = P(A) - P(A \cap B^c) = \frac{1}{5} - \frac{2}{15} = \frac{1}{15}$

$P(A) \cdot P(B) = \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{15}$

Independence: $P(A \cap B) = P(A) \cdot P(B)$

(26)

A_x = "number x is observed at least once"

$$P(A_1 \cap A_6) = ?$$

$$P(A_1) = 1 - P(A_1^c) = 1 - \left(\frac{5}{6}\right)^n$$

$$P(A_6) = 1 - \left(\frac{5}{6}\right)^n$$

$$P(A_1 \cup A_6) = 1 - P(A_1^c \cap A_6^c) = 1 - \left(\frac{4}{6}\right)^n = 1 - \left(\frac{2}{3}\right)^n$$

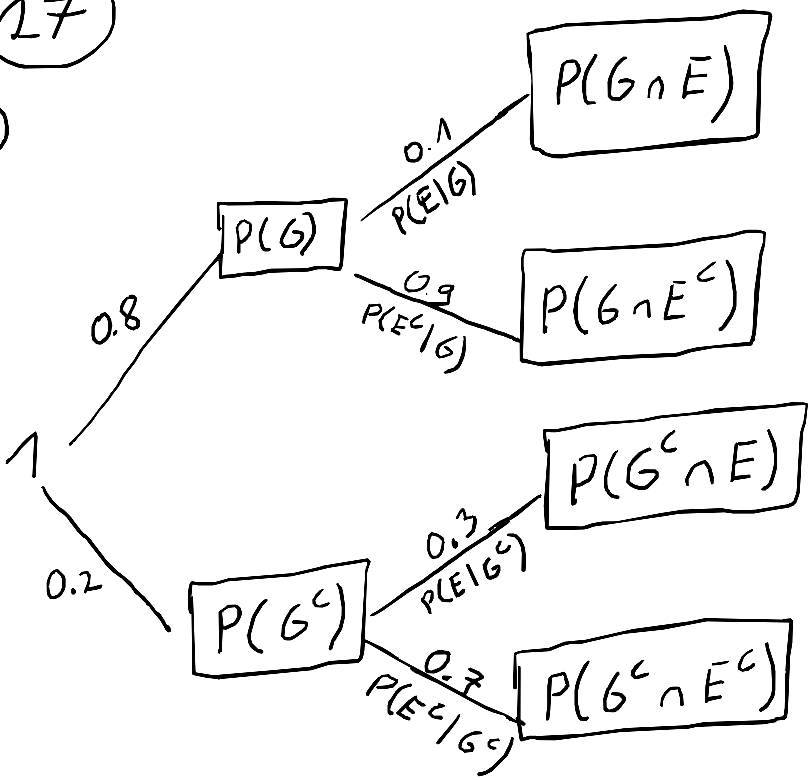
$$P(A_1 \cap A_6) = P(A_1) + P(A_6) - P(A_1 \cup A_6) =$$

$$= 2\left(1 - \left(\frac{5}{6}\right)^n\right) - \left(1 - \left(\frac{2}{3}\right)^n\right) =$$

$$= 2\left(1 - \left(\frac{5}{6}\right)^n\right) - 1 + \left(\frac{2}{3}\right)^n$$

27

a)



b) $P(E) = P(E|G)\cdot P(G) + P(E|G^c)\cdot P(G^c) =$

$= \frac{1}{10}\cdot\frac{8}{10} + \frac{3}{10}\cdot\frac{2}{10} = \frac{8}{100} + \frac{6}{100} = \frac{14}{100} = 14\%$

c) $P(G|E^c) = \frac{P(G\cap E^c)}{P(E^c)} = \frac{\frac{8}{10}\cdot\frac{9}{10}}{\frac{86}{100}} = \frac{72}{86}$

$P(E^c) = 1 - P(E) = \frac{86}{100}$

(28)

$D_x \rightarrow$ "item nr. x is defective"

100 items total
5 defective, 95 good

$$\left[(D_1, D_2^c, D_3^c), (D_1^c, D_2, D_3^c), (D_1^c, D_2^c, D_3)\right]$$

D_1: $\frac{5}{100}$, D_2^c: $\frac{95}{99}$, D_3^c: $\frac{94}{98}$

$$\left(\frac{5 \cdot 95 \cdot 94}{100 \cdot 99 \cdot 98}\right) \cdot 3$$

$$\frac{3 \cdot 5 \cdot 95 \cdot 94}{100 \cdot 99 \cdot 98} = \frac{133950}{970200} =$$

$$= 0.138\ldots \approx 0.14$$

(31) S → "email is a spam"

R → "email contains word refinance"

$P(S) = \frac{1}{2}$ $\qquad\qquad$ $P(S|R) = ?$

$P(R|S) = \frac{1}{100}$

$P(R|S^c) = \frac{1}{100{,}000}$

$$P(S|R) = \frac{P(R|S) \cdot P(S)}{P(R)} =$$

$$= \frac{\frac{1}{100} \cdot \frac{1}{2}}{P(R|S) \cdot P(S) + P(R|S^c) \cdot P(S^c)} =$$

$$= \frac{\frac{1}{100} \cdot \frac{1}{2}}{\frac{1}{100} \cdot \frac{1}{2} + \frac{1}{100{,}000} \cdot \frac{1}{2}} = \frac{\cancel{\frac{1}{2}} \cdot \frac{1}{100}}{\cancel{\frac{1}{2}}\left(\frac{1}{100} + \frac{1}{100{,}000}\right)} =$$

$$= \frac{\frac{1}{100}}{\frac{1001}{100{,}000}} = \frac{1}{100} \cdot \frac{100{,}000}{1001} = \frac{100{,}000}{100{,}100}$$

(34)

$A = \text{"} X = 2 \text{"}$

$B = \text{"} X + Y = 7 \text{"}$

$C = \text{"} Y = 3 \text{"}$

$A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}$

$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$C = \{(1,3), (2,3), (3,3), (4,3), (5,3), (6,3)\}$

a)

$A \cap B = \{(2,5)\}$

$P(A \cap B) = \frac{1}{36}$

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

$P(A) \cdot P(B) = \frac{1}{36} = P(A \cap B)$

INDEPENDENT

b) $A \cap C = \{(2,3)\}$

$P(A \cap C) = \frac{1}{36} = P(A) \cdot P(C)$

INDEPENDENT

c) $B \cap C = \{(4,3)\}$

$P(B \cap C) = \frac{1}{36} = P(B) \cdot P(C)$

INDEPENDENT

d) $A \cap B \cap C = \emptyset$

$P(A \cap B \cap C) = 0 \neq P(A)P(B)P(C)$

NOT INDEPENDENT