VU VRIJE UNIVERSITEIT AMSTERDAM

Probability & Statistics

Final Exam Study Guide

Faculty of Science

Vrije Universiteit Amsterdam

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Contents

Important Information

What You DON'T Need to Memorize

- Formulas, means, and variances for special distributions (binomial, exponential, etc.)
- Formulas for confidence intervals and test-statistics
- Quantiles of distributions (normal, t, chi-squared)
- Chebyshev's inequality

All of these will be provided on the formula sheet!

Calculator Policy: A simple (non-graphical) calculator is allowed.

Textbook Coverage:

- Chapter 1 (except 1.3.5)
- Chapter 3 (except hypergeometric distribution)
- Chapter 4: Sections 4.1 and 4.2 (except method of transformations, memoryless property of exponential, Sections 4.2.4 and 4.2.5)
- Chapter 7.1
- Chapter 8.1 (except order statistics), 8.2 (except 8.2.4), 8.3, 8.4 (except Type II errors, relation to confidence intervals, Section 8.4.5)

1 Basic Concepts

1.1 Review of Set Theory (Section 1.2)

Key Definitions & Properties to Know

Fundamental concepts: set, subset, empty set, universal set, union, intersection, complement, difference, cardinality
Set relationships: mutually exclusive (disjoint), partition, Cartesian product
Countability: countable vs. uncountable sets
Functions: domain, co-domain, range
Important laws:

- De Morgan's laws
- Distributive laws
- Inclusion-exclusion principle (for 2 and 3 sets)

Skills to Master:

- Determine sets involving unions, intersections, complements, differences, and cardinality
- Apply inclusion-exclusion principle for 2 and 3 sets
- Determine range and codomain of functions
- Determine if a set is countable or uncountable

▷ **Lecture Examples**

Examples 1.4, 1.5

▷ **Textbook Problems**

Section 1.2.5: Problem 4
Section 1.5: Exercises 1, 2, 5, 6, 7, 12

1.2 Random Experiments and Probabilities (Section 1.3)

Key Definitions & Properties to Know

- Random experiment
- Probability (measure)
- Discrete probability model
- Geometric series
- Inclusion-exclusion principle for 2 and 3 events

Skills to Master:

- Determine sample spaces of simple experiments
- Calculate probabilities in simple situations

▷ **Lecture Examples**

Examples 1.7, 1.11, 1.12

▷ **Textbook Problems**

Section 1.3.6: Problems 2, 3, 4
Section 1.5: Exercises 13, 14, 16, 17, 18, 22

1.3 Conditional Probability (Section 1.4)

Key Definitions & Properties to Know

- Conditional probability
- Independence (of events)
- Law of total probability
- Bayes' rule

Skills to Master:

- Calculate conditional probabilities
- Determine if given random variables are independent
- Apply independence to solve problems

▷ **Lecture Examples**

Examples 1.15, 1.17, 1.18, 1.19, 1.20, 1.22, 1.24, 1.25, 1.26

▷ **Textbook Problems**

Section 1.4.5: Problems 1, 2
Section 1.5: Problems 25, 28, 30, 34, 35

2 Discrete Random Variables

2.1 Basic Concepts (Section 3.1)

Key Definitions & Properties to Know

- Random variable (continuous/discrete)
- Probability mass function (PMF)
- Independence (of random variables)

Skills to Master:

- Calculate probabilities involving discrete random variables given their PMF
- Determine the PMF of (functions of) discrete random variables

▷ Lecture Examples	▷ Textbook Problems
—	Section 3.1.6: Problems 1, 2, 3, 4, 6, 8 Section 3.2.5: Problem 3

2.2 More About Discrete Random Variables (Section 3.2)

Key Definitions & Properties to Know

- Cumulative distribution function (CDF)
- Expectation and variance
- LOTUS (Law of the Unconscious Statistician)

Skills to Master:

- Determine the CDF of (functions of) discrete random variables
- Calculate expectation and variance of (functions of) discrete random variables

▷ Lecture Examples	▷ Textbook Problems
—	Section 3.2.5: Problems 1, 2, 4, 6, 8, 9

3 Continuous and Mixed Random Variables

3.1 Continuous Random Variables (Section 4.1)

Key Definitions & Properties to Know

- Continuous random variable
- Probability density function (PDF)
- Range
- Expected value and variance

Skills to Master:

- Calculate expected values and variances of (functions of) continuous random variables
- Calculate probabilities involving continuous random variables
- Determine the PDF and CDF of continuous random variables

▷ **Lecture Examples**

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▷ **Textbook Problems**

Section 4.1.4: Problems 1, 2, 3
Section 4.4: Problems 1, 2, 4, 5, 8, 9, 11

3.2 Special Distributions (Section 4.2)

Key Definitions & Properties to Know

- Normal (Gaussian) distribution

Note: Formulas for special distributions will be provided!

Skills to Master:

- Calculate probabilities involving exponential and normal random variables

▷ **Lecture Examples**

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▷ **Textbook Problems**

Section 4.2.6: Problems 3, 4, 5
Section 4.4: Problems 8, 9, 11

4 Limit Theorems and Convergence

Key Definitions & Properties to Know

- Weak Law of Large Numbers (WLLN)
- Central Limit Theorem (CLT)
- Continuity correction

Skills to Master:

- Use the CLT to approximate probabilities
- Use the continuity correction to approximate probabilities

▷ **Lecture Examples**

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▷ **Textbook Problems**

Section 7.3: Problems 2, 3, 4, 5, 6, 7

5 Statistical Inference I: Classical Methods

5.1 Introduction (Section 8.1)

Key Definitions & Properties to Know

- Random sample
- Sample mean

5.2 Point Estimation (Section 8.2)

Key Definitions & Properties to Know

- (Point) estimator
- Bias and unbiased estimator
- Mean squared error (MSE)
- Consistent estimator
- Sample variance and standard deviation
- Likelihood function
- Maximum likelihood estimator (MLE)

Skills to Master:

- Calculate bias and MSE of an estimator
- Determine if an estimator is unbiased
- Calculate the (log) likelihood function from PMF or PDF
- Find maximum likelihood estimators

▷ Lecture Examples	▷ Textbook Problems
—	Section 8.6: Problems 2, 3, 4, 5, 7, 8

5.3 Interval Estimation (Section 8.3)

Key Definitions & Properties to Know

- Interval estimator
- Confidence interval
- Confidence level
- Pivot

Skills to Master:

- Construct confidence intervals for the mean, proportion, and variance based on observed sample quantities (sample mean and variance)

▷ **Lecture Examples**	▷ **Textbook Problems**
—	Section 8.6: Problems 11, 12, 13, 14

5.4 Hypothesis Testing (Section 8.4)

Key Definitions & Properties to Know

- Null and alternative hypothesis
- Test-statistic
- Acceptance and rejection region
- Type I and Type II error
- p-value

Skills to Master:

- Choose an appropriate test-statistic to test a hypothesis about the mean
- Carry out the hypothesis test
- Calculate the p-value
- Draw a conclusion from the test

▷ **Lecture Examples**	▷ **Textbook Problems**
—	Section 8.6: Problems 15, 16, 17, 18, 19, 20

Good luck with your exam preparation!

— Jakub Kuwalek

Study Progress Checklist

Use this checklist to track your progress. Check off each item as you complete it!

Lecture Examples

These examples are from the lecture slides for each topic.

Chapter 1: Basic Concepts

☐ Example 1.4
☐ Example 1.5
☐ Example 1.7
☐ Example 1.11
☐ Example 1.12
☐ Example 1.15
☐ Example 1.17
☐ Example 1.18
☐ Example 1.19
☐ Example 1.20
☐ Example 1.22
☐ Example 1.24
☐ Example 1.25
☐ Example 1.26

Textbook Problems

Chapter 1: Basic Concepts

Section 1.2.5
☐ Problem 4

Section 1.3.6
☐ Problem 2
☐ Problem 3
☐ Problem 4

Section 1.4.5
☐ Problem 1
☐ Problem 2

Section 1.5
☐ Exercise 1
☐ Exercise 2
☐ Exercise 5
☐ Exercise 6
☐ Exercise 7
☐ Exercise 12
☐ Exercise 13
☐ Exercise 14
☐ Exercise 16
☐ Exercise 17
☐ Exercise 18
☐ Exercise 22
☐ Problem 25
☐ Problem 28
☐ Problem 30
☐ Problem 34
☐ Problem 35

Chapter 3: Discrete Random Variables

Section 3.1.6
☐ Problem 1
☐ Problem 2
☐ Problem 3
☐ Problem 4
☐ Problem 6
☐ Problem 8

Section 3.2.5
☐ Problem 1
☐ Problem 2
☐ Problem 3
☐ Problem 4
☐ Problem 6
☐ Problem 8
☐ Problem 9

Chapter 4: Continuous and Mixed Random Variables

Section 4.1.4
☐ Problem 1
☐ Problem 2
☐ Problem 3

Section 4.2.6
☐ Problem 3
☐ Problem 4
☐ Problem 5

Section 4.4
☐ Problem 1
☐ Problem 2
☐ Problem 4
☐ Problem 5
☐ Problem 8
☐ Problem 9
☐ Problem 11

Chapter 7: Limit Theorems

Section 7.3
☐ Problem 2
☐ Problem 3
☐ Problem 4
☐ Problem 5
☐ Problem 6
☐ Problem 7

Chapter 8: Statistical Inference

Section 8.6
☐ Problem 2
☐ Problem 3
☐ Problem 4
☐ Problem 5
☐ Problem 7
☐ Problem 8
☐ Problem 11
☐ Problem 12
☐ Problem 13
☐ Problem 14
☐ Problem 15
☐ Problem 16
☐ Problem 17
☐ Problem 18
☐ Problem 19
☐ Problem 20